

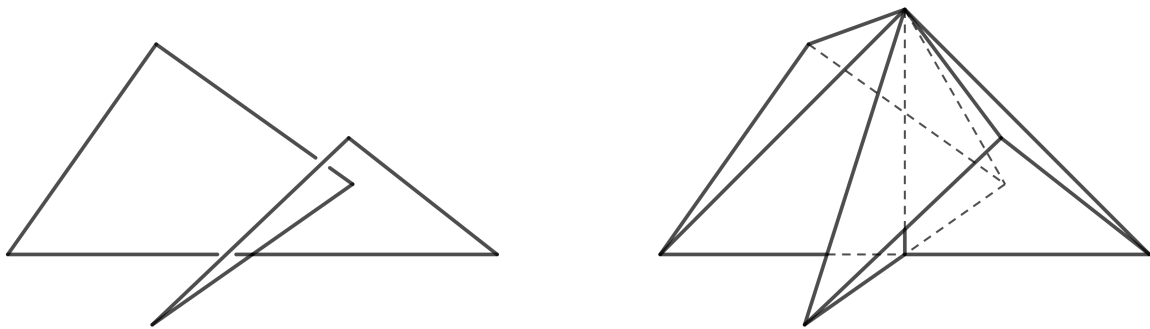
# Using Direct and Constructive Methods for the Existence of Origami Models with Given Boundary Conditions

*R. Geretschläger, S.L. Keeling*

keywords: reconstruction, polyline, triangulation

## Abstract

Whenever a unit square is folded to create an origami in three-dimensional space, the edge of the paper forms a closed curve in space with a total length equal to four units. The first goal of this paper is to show some of the restrictions applicable to such resulting closed curves, specifically in the case of classic origamis, in which none of the sections of the folded paper is curved in any way. (Of course, the resulting curve must be a polyline in this case.) Furthermore, if a polyline fulfilling the required conditions is given, it is of interest to determine origamis whose edges coincide with it. While such an origami will not be unique, if it exists at all, we can expect there to be a simplest resulting origami in some sense. We can see an example of this in Figure 1. The bounding polyline shown on the left yields the simple origami on the right.



**Figure 1:** a bounding polyline and its associated origami model

This origami is certainly not unique, since any sink-fold on the upper peak creates a new origami with the same bounding polyline, but the model shown is completely rigid, as it is composed entirely of triangles, and it is reasonable to consider this a kind of triangulation of the polyline.

Restricting consideration to plane sectors on the resulting origamis, it is possible to apply methods of classic euclidean geometry. This is an alternative to the numerical approach investigated by the authors in [1] for the more general case.

There are numerous restrictions both to possible polylines and to the origamis resulting from such polylines. For instance, not only must the total length of the polyline be equal to 4 units, the fact that the edges of the unit square are at right angles to each other in the corners yields restrictions on the line segments of the polylines that result from these edges. The polyline must therefore consist of four sections, each of unit length, obeying these restrictions. Furthermore, the area of the unit square is equal to one square unit, and this must obviously also be the case for the sum of the areas of the sections of the origami.

The creases of the origami will also have to obey certain laws with respect to the line segments comprising the polyline. A large part of this paper will concern itself with the identification of possible positions of such creases relative to the segments of the polyline.

**Reference:**

[1] S.L.Keeling and R. Geretschläger, *Using Variational and PDE Methods for the Existence of Origami Models with Given Boundary Conditions*, Proceedings of OSME7, 2018.